Set Theory
Quiz 8

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show by transfinite induction that $1+\alpha=\alpha$ for any infinite ordinal α. (Since this is about infinite ordinals, the proof should start with the case $\alpha=\omega$.)

Base case ($\alpha=\omega$):

$$\begin{aligned} 1+\alpha &= 1+\omega \\ &= 1+\textstyle\bigcup_{n<\omega} n \\ &= \textstyle\bigcup_{n<\omega}(1+n) && \text{(Defn. of + when 2nd summand is limit)} \\ &= \omega \\ &= \alpha \end{aligned}$$

Successor case ($\alpha=S(\beta)$):

$$\begin{aligned} 1+\alpha &= 1+S(\beta) \\ &= S(1+\beta) \\ &= S(\beta) && \text{(IH=Inductive Hypothesis)} \\ &= \alpha \end{aligned}$$

Limit case ($\alpha=\bigcup_{\beta<\alpha}\beta$):
(May assume that $\alpha>\omega$, since the $\alpha=\omega$ case has been handled.)

$$\begin{aligned} 1+\alpha &= 1+\textstyle\bigcup_{\beta<\alpha}\beta \\ &= \textstyle\bigcup_{\beta<\alpha}(1+\beta) \\ &= \textstyle\bigcup_{\beta<\omega}(1+\beta)\ \cup\ \bigcup_{\omega\le\beta<\alpha}(1+\beta) \\ &= \omega\cup\textstyle\bigcup_{\omega\le\beta<\alpha}\beta && \text{(IH)} \\ &= \omega\cup\alpha \\ &= \alpha \end{aligned}$$