Set Theory
Quiz 7

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Outline a proof that the class of ordinal numbers is a proper class. (You don't have to justify any statement you make, just list the stages of the proof. Something like "First show Fact X. Then show Fact Y. Then show Fact Z. This is enough, because of reason W.")

- First show that $\alpha \notin \alpha$ for any $\alpha \in \mathrm{ON}$.
- Then show that ON is transitive.
- Then show that ON is well-ordered by $\in$.
- This is enough, since it shows that ON is definable by a formula (hence is a class), but it cannot be a set since then it would be an ordinal. This would lead to $\mathrm{ON} \in \mathrm{ON}$, which is impossible by the first item above.