

## Set Theory

### Quiz 7

Name: \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Outline a proof that the class of ordinal numbers is a proper class. (You don't have to justify any statement you make, just list the stages of the proof. Something like "First show Fact  $X$ . Then show Fact  $Y$ . Then show Fact  $Z$ . This is enough, because of reason  $W$ .")

- First show that  $\alpha \notin \alpha$  for any  $\alpha \in \text{ON}$ .
- Then show that ON is transitive.
- Then show that ON is well-ordered by  $\in$ .
- This is enough, since it shows that ON is definable by a formula (hence is a class), but it cannot be a set since then it would be an ordinal. This would lead to  $\text{ON} \in \text{ON}$ , which is impossible by the first item above.