Set Theory
Quiz 5

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show that if X is finite, then $X \cup \{a\}$ is finite.

(1) Assume first that $a \in X$. If X is finite, then $X \cup \{a\} = X$ is finite too.

(2) Now assume that $a \notin X$. If X is finite, then there is a bijection $f : n \to X$ for some $n \in \omega$.

Claim: the set $g = f \cup \{(n, a)\}$ is a bijection from $S(n)$ to $X \cup \{a\}$.

[g is a function with domain $S(n)$] Since f is a function with domain n, the first coordinates of pairs in f are distinct from each other and n is not among them. Hence the first coordinates of pairs in g are distinct and the set of these elements is $\mathrm{dom}(f) \cup \{n\} = n \cup \{n\} = S(n)$.

[g is injective] We assume that $u \neq v$, but $g(u) = g(v)$, and argue to a contradiction. Since f is 1-1, and $f = g$ on $\mathrm{dom}(f)$, it cannot be that $u, v \in \mathrm{dom}(f)$, so one of these elements (say v) must equal n. The other, u, must be in $\mathrm{dom}(g) - \{v\} = \mathrm{dom}(f)$. Hence $a = g(n) = g(v) = g(u) = f(u) \in X$. But $a \notin X$, so this is a contradiction.

[g is surjective] $\mathrm{ran}(g) = \mathrm{ran}(f) \cup \{a\} = X \cup \{a\}$.