Set Theory
Quiz 4

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show that if $A \subseteq B \subseteq C$ and $|C| = |A|$, then $|A| = |B|$.

The inclusion map $\iota_{AB} : A \to B$, which is injective, witnesses that $|A| \leq |B|$. Composing the inclusion map $\iota_{BC} : B \to C$ with a bijection $b : C \to A$ (which must exist since $|C| = |A|$), we obtain an injective map $b \circ \iota_{BC} : B \to A$, hence $|B| \leq |A|$. From $|A| \leq |B|$ and $|B| \leq |A|$, the CBS Theorem guarantees that $|A| = |B|$.

2. Show that if A is a transitive set, then $A \subseteq \mathcal{P}(A)$.

$$\begin{aligned} A \text{ is transitive} \quad &\Leftrightarrow (C \in B \in A \Rightarrow C \in A) \\ &\Leftrightarrow \text{Every element } B \text{ of } A \text{ is a subset of } A \\ &\Leftrightarrow A \subseteq \mathcal{P}(A) \end{aligned}$$