Logic.

The subject of logic is concerned with the correctness of reasoning. We shall discuss this topic in stages, by addressing the following questions.

(1) What kinds of mathematical objects do we want to study and write about?
(2) What is the syntax of a correctly written statement?
(3) What does it mean for a statement to be true?
(4) What is a proof?
(5) What are some strategies for creating and writing proofs?

Let's begin with an example. Goldbach's Conjecture is the statement that every even number greater than 2 is a sum of two prime numbers. No one knows if Goldbach's Conjecture is true, which is why it is called a "conjecture". Let us write it down formally. Since the statement refers to even numbers, it will help to have a formal expression $\varphi_{\text{even}}(x)$ which is true about x exactly when x is even. Similarly, it will help to have a formal expression $\varphi_{\text{prime}}(x)$ which is true about x exactly when x is prime. If we had such expressions, then Goldbach's Conjecture could be written

$$(\forall x)(((x > 2) \wedge \varphi_{\text{even}}(x)) \to (\exists y)(\exists z)(\varphi_{\text{prime}}(y) \wedge \varphi_{\text{prime}}(z) \wedge (x = y + z))).$$

I would read this out loud by saying "For all x, if x is greater than 2 and satisfies a formula expressing that x is even, then there exists y and z, each satisfying a formula expressing that they are prime, such that x equals y plus z". In this formal statement $\varphi_{\text{even}}(x)$ is an abbreviation for something like

$$(\exists w)(x = w + w),$$

Which I would read as "There exists w such that x equals w plus w". $\varphi_{\text{prime}}(x)$ is an abbreviation for something like

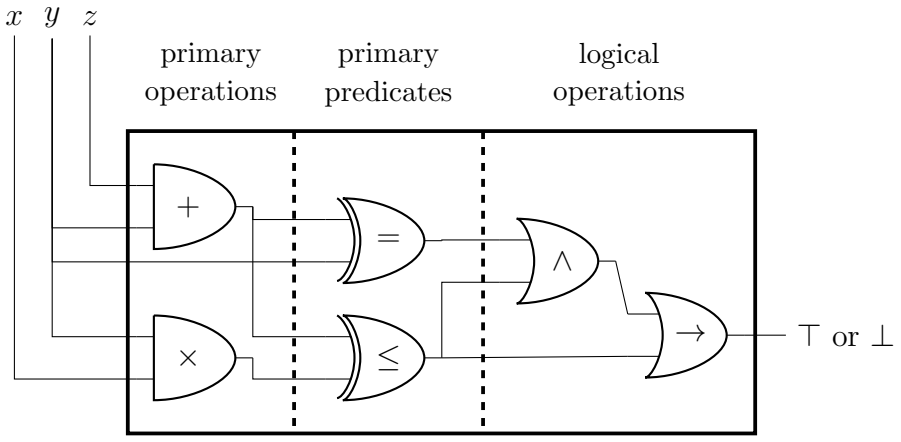
$$(x > 1) \wedge (\forall u)((\exists v)(x = uv) \to ((u = 1) \vee (u = x))),$$

which I would read as "x is greater that 1 and for all u, if there exists v such that $x = uv$, then u equals 1 or x".

One purpose for expressing Goldbach's Conjecture formally is to make the underlying structure of the sentence clear.

Predicates.

the part of a sentence or clause containing a verb and stating something about the subject



compound predicate