

The Axioms of Set Theory.

Equality

- (1) (Extensionality) Two sets are equal if they have the same elements.

Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.
(Call it the *empty set* and denote it by $\emptyset, \{ \}$ or 0.)
- (3) (Infinity) There is an inductive set.

Creation of New Sets

- (4) (Pairing) If A and B are sets, then there is a set $\{A, B\}$ whose elements are exactly A and B .
- (5) (Union) If A is a set, then there is a set $\bigcup A$ whose elements are exactly the elements of the elements of A .
- (6) (Power Set) If A is a set, then there is a set $\mathcal{P}(A)$ whose elements are exactly the subsets of A .
- (7) (Comprehension) If A is a set and P is a property given by a formula, then there is a set $\{x \in A \mid P(x)\}$ whose elements are exactly the elements of A that satisfy P .
- (8) (Replacement) If A is a set and F is a function given by a formula, then there is a set $\{F(x) \mid x \in A\}$ whose elements are exactly the F -values of elements of A .
- (9) (Choice) If $\{A_i \mid i \in I\}$ is set of nonempty disjoint sets, then there is a set C such that $|A_i \cap C| = 1$ for every i .

Sets have Special Properties

- (10) (Foundation) If A is a nonempty set, then there is a set $B \in A$ such that $A \cap B = \emptyset$.
(This axiom allows us to assign an ordinal rank or “creation stage” to any set, so that every set has larger rank than its elements.)