ANALYSIS 1
MIDTERM

You have 50 minutes for this exam. You may not use any unauthorized sources, and you may not communicate with others about the exam. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

I have neither given nor received aid on this exam.

Name:______________________________

1. Define the given word or phrase. Use complete sentences.

(a) *supremum* (of a set of real numbers).

The *supremum* of a subset $A \subseteq \mathbb{R}$ is its least upper bound.

(b) *convergent sequence*.

The sequence $(a_i)_{i\in\mathbb{N}}$ is *convergent* if the formal sentence

$$(\exists L)(\forall \varepsilon > 0)(\exists N)(\forall i)((i > N) \to (|a_i - L| < \varepsilon))$$

is true in the structure $\mathbb{R} = \langle \{\text{reals}\}; +, -, 0, \cdot, 1, < \rangle$.

2. State the theorems. Include all necessary hypotheses.

(a) Cantor-Bernstein-Schröder Theorem.

Assume that A, B are sets. If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.

(b) The Monotone Convergence Theorem.

A monotone bounded sequence is convergenet.

3. Determine the truth value of the following formal sentence, and justify your answer by giving a winning strategy for the appropriate quantifier.

(a) Is $\forall x\ \exists y\ \exists z((y < x) \wedge (x < z))$ true in $\mathbb{R}$?

Truth Value: True

Appropriate quantifier: $\exists$

- $\forall$ chooses some x
- $\exists$ chooses $y = x - 1$
- $\exists$ chooses $z = x + 1$

(b) Is the formal sentence $(\forall L)(\exists \varepsilon > 0)(\forall N)(\exists i)((i > N) \wedge (|1 - L| \not< \varepsilon))$ true in $\mathbb{R}$? (In other words, is $(1, 1, 1, \ldots)$ divergent?)

Truth Value: False

Appropriate quantifier: $\forall$

- $\forall$ chooses $L = 1$
- $\exists$ chooses some ε
- $\forall$ chooses $N = 1$
- $\exists$ chooses some i

4. Write formal sentences:

(a) Write a formal sentence that is meaningful for any ordered field, and is true in $\mathbb{Q}$ but false in $\mathbb{R}$.

$$(\forall x)(x^2 \neq 2)$$

(b) Write a formal sentence that expresses that the sequence $(a_i)_{i \in \mathbb{N}}$ diverges.

$$(\forall L)(\exists \varepsilon > 0)(\forall N)(\exists i)((i > N) \wedge (|a_i - L| \not< \varepsilon))$$