

Analysis 1
Quiz 9

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. What is a metric?

A metric on a set A is a function $d : A \times A \rightarrow \mathbb{R}$ satisfying

- (Positive definiteness) $d(a, b) \geq 0$, with equality iff $a = b$.
- (Symmetry) $d(a, b) = d(b, a)$
- (Triangle inequality) $d(a, c) \leq d(a, b) + d(b, c)$.

2. If $\langle X; d \rangle$ is a metric space, how do you determine whether a subset $U \subseteq X$ is open in the metric topology?

$U \subseteq X$ is open if for every $p \in U$ there is an open ball $B(p, \varepsilon)$ such that $B(p, \varepsilon) \subseteq U$.

To make the quantifier structure clear, you might elect to write this as

$$(\forall p \in U)(\exists \varepsilon > 0)(B(p, \varepsilon) \subseteq U).$$