

Analysis 1  
Quiz 7

Name: \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. State the following. (Include all necessary hypotheses.)

(a) The Cauchy Criterion for Series.

A series  $\sum_{n=1}^{\infty} a_n$  converges if and only if  
 $(\forall \varepsilon > 0)(\exists N)(\forall i)(\forall j)((j > i > N) \rightarrow (|a_{i+1} + \cdots + a_j| < \varepsilon))$   
holds in  $\mathbb{R}$ .

(b) The Comparison Test.

Assume  $(a_n)$  and  $(b_n)$  are sequences satisfying  $0 \leq a_n \leq b_n$  for all  $n \in \mathbb{N}$ .  
(i) If  $\sum_{n=1}^{\infty} b_n$  converges, then  $\sum_{n=1}^{\infty} a_n$  converges.  
(ii) If  $\sum_{n=1}^{\infty} a_n$  diverges, then  $\sum_{n=1}^{\infty} b_n$  diverges.

(c) The Absolute Convergence Test.

If  $\sum a_n$  converges absolutely, then it converges.

(d) Dirichlet's Test.

If the partial sums of  $\sum b_n$  are bounded and  $(a_k)$  is a monotone nonincreasing sequence with  $\lim_{n \rightarrow \infty} a_n = 0$ , then the series  $\sum_{n=1}^{\infty} a_n b_n$  converges.