Analysis 1
Quiz 5

Name:____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Explain why the Monotone Convergence Theorem guarantees that the following sequence converges: $(a_n)_{n\in\mathbb{N}} = (1,\ 0.1,\ 0.01,\ 0.001, \ldots)$. (The general term is $a_n = 10^{-n} = 1/10^n$.)

For the MCT to guarantee that this sequence converges it is enough to observe that this sequence is monotone decreasing and bounded below by 0.

2. Let $(a_n)_{n\in\mathbb{N}}$ be the sequence from Problem 1. Show that $\lim_{n\to\infty} a_n = 0$ by giving a winning strategy for the appropriate quantifier in the game defined by "$(\forall\varepsilon > 0)(\exists N)(\forall i)((i > N) \to (|a_i - 0| < \varepsilon))$".

Winning strategy:

- $\forall$ chooses some ε, which we may assume satisfies $\varepsilon > 0$.
- $\exists$ chooses N so that $10^{-N} < \varepsilon$, or equivalently $10^N > 1/\varepsilon$, or equivalently $N > \log_{10}(1/\varepsilon)$.
- $\forall$ chooses some i, which we may assume satisfies $i > N$.

This is a winning strategy for $\exists$, since if $i > N$, then $i > \log_{10}(1/\varepsilon)$, so $10^i > 1/\varepsilon$, so $a_i = 10^{-i} < \varepsilon$, so $|a_i - 0| < \varepsilon$.