

Analysis 1
Quiz 4

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Write down the formal sentence which defines “ $\lim_{i \rightarrow \infty} a_i = L$ ”. (Your sentence should not contain English words.)

$$(\forall \varepsilon > 0)(\exists N)(\forall i) \left((i > N) \rightarrow (|a_i - L| < \varepsilon) \right)$$

2. Explain why $\lim_{i \rightarrow \infty} \frac{1}{i} = 0$. (Use your answer from Part 1.)

We need to produce a winning strategy for \exists for the game

$$(\forall \varepsilon > 0)(\exists N)(\forall i) \left((i > N) \rightarrow \left(\left| \frac{1}{i} \right| < \varepsilon \right) \right).$$

Strategy for \exists :

- \forall chooses some ε . (\exists can't control this choice, but we may assume at least that $\varepsilon > 0$.)
- \exists chooses $N > 1/\varepsilon$. (This is the important element of the solution!)
- \forall chooses some i . (\exists can't control this choice, but we may assume at least that $i > N$.)

This is a winning strategy, since if $i > N$ and $N > 1/\varepsilon$, then $1/i < 1/N < \varepsilon$. Also $0 < 1/i$, so $1/|i| < \varepsilon$.