**Analysis 1**
**Quiz 10**

**Name:**____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Our book's version of the Heine-Borel Theorem characterizes compactness of subsets $K \subseteq \mathbb{R}$ in 3 ways: one way involving sequences, one way involving the metric topology, and one way that is purely topological (that is, which refers only to properties of open sets). Write each of these characterizing statements using complete sentences.

(a) (Compactness in terms of sequences)

Every sequence of elements of $K$ has a convergent subsequence, and any such subsequence converges to an element of $K$.

(b) (Compactness in terms of the metric)

$K$ is closed and bounded.

(c) (Compactness in terms of the topology)

Every open cover of $K$ has a finite subcover.

2. Using one of the characterizations of compactness, show that $\mathbb{R}$ is not compact.

Any one of the following answers suffices.

(a) Let $a_n = n$. Then $(a_n)_{n \in \mathbb{N}}$ is a sequence in $\mathbb{R}$ that has no convergent subsequence, so $\mathbb{R}$ is not compact.

(b) $\mathbb{R}$ is not bounded, since there is no ball $B(a,r) = (a-r, a+r)$ that contains all real numbers. Hence $\mathbb{R}$ is not compact.

(c) $\mathbb{R}$ can be covered by the set of all balls $B(a,1)$ of radius 1. No finite subcollection can cover $\mathbb{R}$, so $\mathbb{R}$ is not compact.