Practice with induction!

The goal is to use induction (mostly!) to prove that $\langle \mathbb{N}; < \rangle$ is a linearly ordered set. Here $m < n$ is defined to mean $m \in n$, and $m \leq n$ is defined to mean $m < n$ or $m = n$.

Prove each statement.

(1) $\forall n(0 \leq n)$. (Use induction on n.)

(2) $m < S(n)$ holds iff $m < n$ or $m = n$. (No induction needed.)

(3) For all a, b, c, if $a < b$ and $b < c$, then $a < c$. (Induction on c.)

(4) $\forall n(n \not< n)$. (Induction on ?.)

(5) $(\forall m)(\forall n)((m < n) \to (S(m) < S(n))$. (Induction on ?.)

(6) $(\forall m)(\forall n)((m < n) \vee (m = n) \vee (n < m))$. (Induction on ?.)