

Some statements equivalent to the Axiom of Choice. ($\text{ZF} + \text{Statement} \equiv \text{ZFC}$)

- (1) (AoC) Every set of nonempty disjoint sets has a choice set.
- (2) (AoC)' Every set of nonempty sets has a choice function.
- (3) (AoC)" For any nonempty set A , $\mathcal{P}(A) \setminus \{\emptyset\}$ has a choice function.
- (4) Every surjection has a right inverse.
- (5) A Cartesian product of nonempty sets is nonempty.
- (6) (Well-Ordering Theorem) Any set can be well-ordered.
- (7) For any two sets A and B , either $|A| \leq |B|$ or $|B| \leq |A|$.
- (8) (Tarski's Theorem) For any infinite set B , $|B \times B| = |B|$.
- (9) (Zorn's Lemma) Any inductively ordered set has a maximal element.
- (10) (Tychonoff's Theorem) A product of compact spaces is compact.
- (11) Every nontrivial ring with unit has a maximal ideal.
- (12) Every vector space has a basis.
- (13) Every set supports a group structure.
- (14) Every connected graph has a spanning tree.
- (15) (Löwenheim-Skolem-Tarski) Any first-order sentence that can be satisfied by at least one infinite structure can be satisfied by at least one structure of any prescribed infinite cardinality.