Practice!

(1) State the theorem.
  (a) Cantor's Theorem.
  (b) Cantor-Bernstein-Schroeder Theorem.
  (c) Monotone Convergence Theorem.
  (d) Bolzano-Weierstrass Theorem.
  (e) Cauchy Criterion.

(2) Define the terms.
  (a) null sequence.
  (b) uncountable set
  (c) infimum
  (d) sequence
  (e) divergent sequence
  (f) monotone sequence
  (g) Cauchy sequence
  (h) infinite series
  (i) harmonic series

(3) True or False? If False, explain why.
  (a) It is possible to prove that uncountable sets exist.
  (b) $\mathbb{Z}$ is an ordered field.
  (c) Every Archimedean ordered field is complete.
  (d) If (a_i) and (b_i) are both bounded, then $(a_i + b_i)$ is also bounded.
  (e) If (a_i) and (b_i) are both convergent, then $(a_i b_i)$ is also convergent.
  (f) If (a_i) and (b_i) are both null, then $(a_i b_i)$ is also null.

(4) Explain why the empty set does not have a supremum.

(5) Give an example of a nonempty set that does not have a supremum.

(6) If (a_i) is an unbounded sequence, can it have a convergent subsequence? Why or why not?

(7) Assume that $a_i \geq 0$ for all i. Show that Σa_i converges iff its partial sums are bounded.

(8) What property of $(a_i)_{i\in\mathbb{N}^*}$ is being expressed by the following sentence?

$$\exists M \; \forall i \; (-M < a_i < M)$$

(9) What property of $(a_i)_{i\in\mathbb{N}^*}$ is being expressed by the following sentence?

$$(\forall L)(\exists \varepsilon > 0)(\forall N)(\exists i)((i > N) \wedge (|a_i - L| \not< \varepsilon))$$

(10) Write a formal sentence that distinguishes between $\langle \mathbb{N}; +, \cdot \rangle$ and $\langle \mathbb{N}^*; +, \cdot \rangle$.

(11) You are given hypotheses of the form $\forall x(A(x) \to B(x))$ and $\forall x(B(x) \to C(x))$. Explain how to prove $\forall x(A(x) \to C(x))$ from these hypotheses, if possible. Else show that it is impossible through a specific example.

(12) You are given hypotheses of the form $\exists x(A(x) \to B(x))$ and $\exists x(B(x) \to C(x))$. Explain how to prove $\exists x(A(x) \to C(x))$ from these hypotheses, if possible. Else show that it is impossible through a specific example.

(13) Show that a convergent sequence has an infinite monotone subsequence. (Hint: Assume $(a_i) \to L$. Argue that if infinitely many terms of (a_i) are smaller than L, then (a_i) has a monotone increasing subsequence. Then consider the case where infinitely many terms of (a_i) are equal L, and the case where infinitely many terms of (a_i) are greater than L.)

(14) Suppose that (a_n) and $((-1)^n a_n)$ both converge. What can you say about $\lim a_n$?

(15) Suppose that the nth partial sum of $\sum a_i$ is $1 - 1/n$. What is a_i? What is $\sum a_i$?

(16) The Supreme's were a 1960's singing group with three female singers. If you put -um at the end of each of their names, they could have been called the Supremums. Give the three names with -um at the end of each name.