

Practice with limits!

If you want to say that a sequence $(a_i)_{i \in \mathbb{N}}$ converges to the limit L , you write that

$$(\forall \varepsilon > 0) (\exists N) (\forall i) ((i > N) \rightarrow (|a_i - L| < \varepsilon)).$$

In the case $L = 5$, this reads

$$(\forall \varepsilon > 0) (\exists N) (\forall i) ((i > N) \rightarrow (|a_i - 5| < \varepsilon)).$$

Now, if you want to say that the sequence $(a_i)_{i \in \mathbb{N}}$ converges to *some* limit, without specifying what the limit is, you write

$$(\exists L) (\forall \varepsilon > 0) (\exists N) (\forall i) ((i > N) \rightarrow (|a_i - L| < \varepsilon)).$$

Finally, if you want to say that the sequence $(a_i)_{i \in \mathbb{N}}$ does not converge, then you want to negate the previous displayed line. You get

$$(\forall L) (\exists \varepsilon > 0) (\forall N) (\exists i) ((i > N) \wedge (|a_i - L| \not< \varepsilon)).$$

Practice!

(1) Show that the sequence $(\frac{1}{i})_{i \in \mathbb{N}^*} = (1, \frac{1}{2}, \frac{1}{3}, \dots)$ converges.

(2) Show that the sequence $(i)_{i \in \mathbb{N}^*} = (1, 2, 3, \dots)$ does not converge.