

## The Axioms of Set Theory.

### Equality

- (1) (Extensionality) Two sets are equal if they have the same elements.

### Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.  
(Call it the *empty set* and denote it by  $\emptyset$ ,  $\{ \}$  or 0.)
- (3) (Infinity) There is an inductive set.

### Creation of New Sets

- (4) (Pairing) If  $A$  and  $B$  are sets, then  $\{A, B\}$  is a set.
- (5) (Union) If  $I$  is a set, and  $A_i$  is a set for each  $i \in I$ , then  $\bigcup_{i \in I} A_i$  is a set.
- (6) (Power Set) If  $A$  is a set, then  $\mathcal{P}(A)$  is a set.
- (7) (Comprehension) If  $A$  is a set and  $P$  is a property given by a formula, then  $\{x \in A \mid P(x)\}$  is a set.
- (8) (Replacement) If  $A$  is a set and  $F$  is a function given by a formula, then  $\{F(x) \mid x \in A\}$  is a set.
- (9) (Choice) If  $\{A_i \mid i \in I\}$  is set of nonempty disjoint sets, then there is a set  $C$  such that  $|A_i \cap C| = 1$  for every  $i$ .

### Sets have Special Properties

- (10) (Foundation) If  $A$  is a nonempty set, then there is a set  $B \in A$  such that  $A \cap B = \emptyset$ .  
(This axiom allows us to assign an ordinal rank or “creation stage” to any set, so that every set has larger rank than its elements.)