ANALYSIS 1 (MATH 3001): REVIEW SHEET 2

Final Exam: Dec 18, 4:30-7pm

VI. Series. (continued)
- (c) Criteria for convergence of series.
- (d) Absolute convergence.
- (e) Riemann Rearrangement Theorem.
- (f) A power series converges absolutely within its radius of convergence.

VII. Topology.
- (a) Definitions: topology/topological space, open and closed sets, interior and closure, neighborhood, limit point, boundary point, interior point, isolated point.
- (b) Examples: discrete topology, trivial topology, cofinite topology.
- (c) Metric/metric topology, ε-ball.
- (d) Cantor set.
- (e) Compact sets.
- (f) Heine-Borel Theorem (characterizing compact subsets of $\mathbb{R}$ in terms of limits, in terms of the metric topology, and in pure topological terms).
- (g) The connected subsets of $\mathbb{R}$ are the intervals.

VIII. Continuous functions.
- (a) Definitions (in terms of limits, in terms of the metric topology, and in pure topological terms).
- (b) Examples of Dirichlet and Thomae.
- (c) Algebraic Continuity Theorem.
- (d) The continuous image of a compact set is compact.
- (e) Heine-Cantor Theorem (a continuous function on a compact set is uniformly continuous).
- (f) The continuous image of a connected set is connected.
- (g) Extreme Value Theorem.
- (h) Intermediate Value Theorem.

IX. Derivatives.
- (a) Definition.
- (b) Differentiability implies continuity.
- (c) Algebraic Differentiability Theorem.
- (d) Interior Extremum Theorem.
- (e) Darboux's Theorem. (Derivatives have the IVP.)
- (f) Rolle's Theorem.
- (g) Mean Value Theorem.
- (h) If $f' > 0$ on $[a, b]$, then f is monotone increasing on $[a, b]$. If $f' < 0$ on $[a, b]$, then f is monotone decreasing on $[a, b]$.

(i) If $f' = 0$ on $[a, b]$, then f is constant on $[a, b]$. If $f' = g'$ on $[a, b]$ and $f(c) = g(c)$ for some $c \in [a, b]$, then $f = g$ on $[a, b]$.
(j) The blancmange function is continuous and periodic, but nowhere differentiable.

X. Sequences and series of functions.
(a) Pointwise versus uniform convergence. Absolute convergence. A pointwise limit of continuous functions need not be continuous.
(b) Uniform Limit Theorem.
(c) Weierstrass M-test.
(d) If (f_n) is a sequence of functions differentiable on $[a, b]$, $\sum f_n$ converges uniformly on $[a, b]$, and there is a $c \in [a, b]$ such that $\sum f_n(c)$ converges, then $\sum f_n$ is differentiable and $(\sum f_n)' = \sum f_n'$.

XI. Integration.
(a) Definition of Riemann integral for bounded functions on closed, bounded intervals.
(b) Terminology for partitions: cells, finer, coarser, refinement.
(c) Integrability Criterion: $(\forall \varepsilon > 0)(\exists P)(U(f, P) - L(f, P) < \varepsilon)$. Also should know that $U(f, P) - L(f, P) = \sum_{k=1}^{\infty} (M_k - m_k)(x_k - x_{k-1})$.
(d) Continuous functions on bounded intervals are integrable.
(e) Algebraic Integrability Theorem.
(f) $\lim_{n\to\infty} \int_a^b f_n = \int_a^b f$ if $(f_n) \to f$ uniformly on $[a, b]$.
(g) Statements of Clairaut's Theorem, Leibniz Integral Rule, and Fubini's Theorem.
(h) Statement of Dini's Theorem.
(i) Measure zero. Oscillation.
(j) Lebesgue's Theorem characterizing the Riemann integrable functions.

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.