

ANALYSIS 1 (MATH 3001): REVIEW SHEET

I. Discrete math review

- (a) Mathematics is intended to be well founded. (Understand the relation of primitivity with respect to definitions and theorems.)
- (b) Reading, writing, and interpreting formal sentences.
- (c) Axioms 1-7 of set theory.
- (d) Terminology for functions.

II. Development of number systems

- (a) How is \mathbb{N} defined?
- (b) How is \mathbb{Z} constructed from \mathbb{N} ?
- (c) How is \mathbb{Q} constructed from \mathbb{Z} ?
- (d) How is \mathbb{R} constructed from \mathbb{Q} ?

III. Ordered fields and the Axiom of Completeness (AoC)

- (a) Axioms for ordered fields, and immediate consequences.
- (b) AoC implies its dual statement.
- (c) AoC implies the Nested Interval Property.
- (d) AoC implies \mathbb{R} is Archimedean.
- (e) \mathbb{Q} is dense within any Archimedean ordered field.

IV. Review of Cardinality

- (a) Definitions of $|A| \leq |B|$, $|A| = |B|$, $|A| < |B|$, equipotence.
- (b) Definitions of ordinal numbers, cardinal numbers, countable versus uncountable.
- (c) Cantor's Theorem, Cantor-Bernstein-Schroeder Theorem, the theorem asserting $|A \times A| = |A|$ for infinite A , and the theorem asserting that a countable union of countable sets is countable.
- (d) $|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}| < |\mathcal{P}(\mathbb{N})| = |\mathbb{R}| = |\mathbb{C}|$.
- (e) Continuum Hypothesis.

V. Sequences.

- (a) Definition of sequence. Definition of subsequence.
- (b) Definition of convergent/divergent sequence.
- (c) Limit Theorems:
 - (i) Algebraic Limit Theorem.
 - (ii) Order Limit Theorem.
 - (iii) Monotone Convergence Theorem.
 - (iv) Bolzano-Weierstrass Theorem.
 - (v) Cauchy Criterion.
- (d) Criteria for divergence.

VI. Series.

- (a) Definition of series.
- (b) Definition of convergent/divergent series.

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.