Analysis 1
Quiz 9

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. *TRUE* or *FALSE*? Justify your answer.

   (a) If $f\colon [a,b] \to \mathbb{R}$ is continuous and attains its maximum at a point $c \in (a,b)$, then $f'(c) = 0$.

FALSE. $f(x) = -|x|$ is continuous on $[-1,1]$ and attains its maximum at $c = 0$, but $f'(0)$ does not exist.

   (b) If a function $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable and its derivative is not constant, then there exists c such that $f'(c)$ is rational.

TRUE. Since f' is nonconstant, there exist $a < b$ such that $f'(a) \neq f'(b)$. Pick any rational number q between $f'(a)$ and $f'(b)$ (which is possible, since the rationals are dense in $\mathbb{R}$), and use Darboux's Theorem to produce a c between a and b such that $f'(c) = q \in \mathbb{Q}$.

2. Recall that the blancmange function is defined by $B(x) = \sum_{k=0}^{\infty} \frac{1}{2^k} h(2^k x)$, where $h(x) = \inf\{|x-n| \mid n \in \mathbb{Z}\}$ is the sawtooth function. Identify those properties of $B(x)$ which allow us to deduce that $B(x)$ is uniformly continuous on $\mathbb{R}$.

$B(x)$ is continuous and periodic.