Analysis 1
Quiz 7

Name:____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Define what it means for a function $f : \mathbb{R} \to \mathbb{R}$ to be continuous.

   f is continuous if $f(\lim a_i) = \lim f(a_i)$ for every convergent sequence $(a_i)_{i \in \mathbb{N}^*}$.

   OR, f is continuous if $(\forall L)(\forall \epsilon > 0)(\exists \delta > 0)(\forall x)(|x - L| < \delta \to |f(x) - f(L)| < \epsilon)$.

   OR, f is continuous if $f^{-1}(O)$ is open for all open sets O.

2. Explain why

   (a) there is a continuous function $f : \mathbb{R} \to \mathbb{R}$ that maps the interval $(0, 2\pi)$ onto the interval $[-1, 1]$ (that is, $f((0, 2\pi)) = [-1, 1]$), but

   $f(x) = \sin(x)$.

   (b) there is no continuous function $f : \mathbb{R} \to \mathbb{R}$ that maps the interval $[-1, 1]$ onto the interval $(0, 2\pi)$ (that is, $f([-1, 1]) = (0, 2\pi)$).

   The continuous image of a compact set is compact. Since $(0, 2\pi)$ is not compact, it cannot be the continuous image of the compact set $[-1, 1]$.