

Analysis 1
Quiz 2

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Is the ordered field of rational numbers, \mathbb{Q} , Archimedean? Explain why or why not.

Yes. It suffices to observe that $\mathbb{Q} = \bigcup_{i=1}^{\infty} [-n, n]$.

2. Show that if $|\mathbb{R}| \leq |X|$, then X is uncountable.

We argue by contradiction.

If X is not uncountable, it must be finite or countably infinite. In this case, $|X| \leq |\mathbb{N}|$. Since $|\mathbb{N}| < |\mathbb{R}|$, this yields $|X| \leq |\mathbb{N}| < |\mathbb{R}| \leq |X|$, hence $|X| = |\mathbb{N}| = |\mathbb{R}|$ by the Cantor-Bernstein-Schröder Theorem. But now we have both $|\mathbb{N}| = |\mathbb{R}|$ and $|\mathbb{N}| < |\mathbb{R}|$, which is impossible.