

## Analysis 1

### Quiz 1

Name: \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. What is the definition of a “null sequence”?

The sequence  $(q_i)_{i \in \mathbb{N}}$  is null if

$$\forall n \exists N \forall i \left( (i > N) \rightarrow \left( |q_i| < \frac{1}{n} \right) \right).$$

2. Use the definition from Problem 1 to show that the sequence  $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots)$  is a null sequence. Your answer should be a description of a winning strategy for  $\exists$ .

The sequence of this question is  $(q_i)_{i \in \mathbb{N}}$  where  $q_i = \frac{1}{i+1}$ .

Here is a winning strategy for  $\exists$ .

- (1)  $\forall$  chooses some value for  $n$ . (We can make no assumptions about this choice, if we are explaining a strategy for  $\exists$ . We have to allow  $\forall$  to make smart moves or dumb moves.)
- (2) The strategy we choose instructs  $\exists$  to choose  $N = n$ .
- (3) Now  $\forall$  chooses some  $i$ .
- (4) Our task is to show that  $\exists$  wins, namely that  $(i > N) \rightarrow (|q_i| < \frac{1}{n})$  is true.

If  $i \not> N$ , then the sentence is true.

It  $i > N = n$ , then  $i + 1 > n + 1$ , so  $\frac{1}{i+1} < \frac{1}{n+1}$ , so

$$|q_i| = \frac{1}{i+1} < \frac{1}{n+1} < \frac{1}{n}.$$

This verifies that  $(i > N) \rightarrow (|q_i| < \frac{1}{n})$  will hold whenever  $\exists$  follows the strategy of choosing  $N = n$ .