

Analysis 1
Quiz 10

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Write a formal sentence which expresses that $(f_n(x))_{n \in \mathbb{N}}$ converges uniformly to $f(x)$.

$$(\forall \varepsilon > 0)(\exists N)(\forall c)(\forall i)((i > N) \rightarrow (|f_i(c) - f(c)| < \varepsilon))$$

2. Let $f_n(x) = x^n/n!$. Apply the Weierstrass M -test to show that $\sum_{n=0}^{\infty} f_n(x)$ converges uniformly on $[-1, 1]$.

We have $|f_n(x)| = |x|^n/n! \leq 1/n!$ on $[-1, 1]$, so take $M_n = 1/n!$. Since $\sum M_n = \sum 1/n!$ converges,¹ we get that $\sum_{n=0}^{\infty} f_n(x)$ converges uniformly on $[-1, 1]$.

¹To explain why $\sum 1/n!$ converges, can use the Ratio Test, or comparison with a geometric series, or just use that it is well known that $\sum 1/n! = e$.