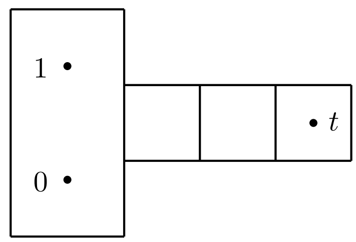
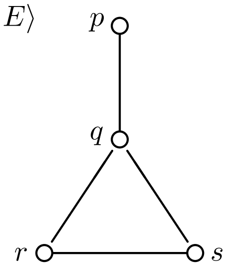
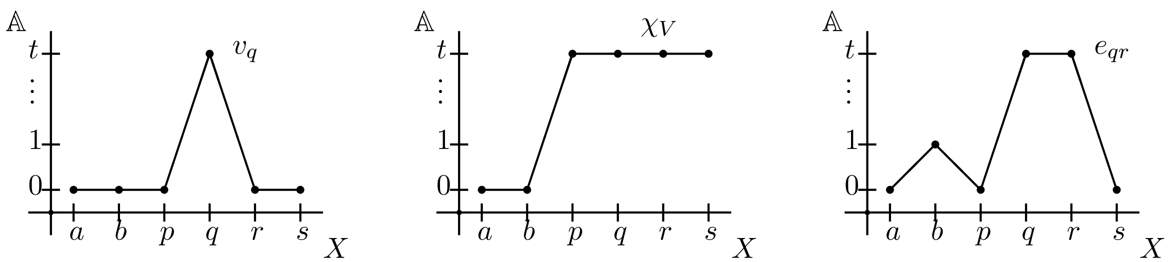
$\mathbb{A} = \mathbb{A}_A$



$G = \langle V; E \rangle$



$p(x,y) =$ pseudomeet
$q(x,y) =$ pseudojoin
$U = e(A) = \{0, t\}$, $e^2 = e$



$$A[G] := \mathrm{Sg}^{\mathbb{A}^X}(\{v_x \mid x \in V\} \cup \{e_{xy} \mid xy \in E\}) \quad (\ni \chi_V)$$

Idea

(1) Show that the BA $\{0,t\}^X|_{\mathbb{A}[G]}$ is definable.
(2) Show that the atoms below χ_V of this BA are the encodings of vertices, and this set is definable by $\mathsf{Atoms}(x)$.
(3) Show that x is adjacent to y in G iff $\exists g \in \mathbb{A}[G]$ that agrees with e_{xy} on G and satisfies $g(a) \neq g(b)$.
(4) Let $S = \{u \in A \mid q(u,t) = t\}$ (definable). Show that the set of S-valued tuples in $\mathbb{A}[G]$ is definable, and contains χ_V and all encodings of vertices and edges.
(5) Show that the set of all g as in (3) is definable within the set of S-valued functions. (Need auxilary formulas $\varphi_S(x)$, $\mathsf{Equal}_{a,b}(x)$, $\mathsf{Good}(x)$.)

Reductions: Assume that $\mathbb{A}$ is a minimal counterexample to the claim that whenever

- $\mathbb{A}$ has a type **3** (Boolean type) $\langle \nu, \mu \rangle$-minimal set with a tail, then
- the class of finite graphs is interpretable into the class of subalgebras of finite powers of $\mathbb{A}$.

Then we may assume that

(1) $\nu = 0$, so $0 \prec \mu$.
(2) Every nonzero congruence of $\mathbb{A}$ is above μ ($\mathbb{A}$ is subdirectly irreducible).
(3) May assume that $\mathbb{A} = \mathbb{A}_A$ and that $\mathbb{A} = B \cup T = N \cup T$ is $\langle 0, \mu \rangle$-minimal of type **3**
(4) There is some $t \in T$ and an idempotent unary polynomial $e(x) = e^2(x)$ such that $U = e(A) = \{0, t\}$. Moreover, $\mathbb{A}|_U$ has the polynomials of a 2-element BA, written $x \vee y$, $x \wedge y$, x'.