

Classification results about minimal sets, irreducible sets, covers.

Minimal sets and irreducible sets.

- (1) Palfy (1984): classified up to polynomial equivalence all finite $\langle =, A \times A \rangle$ -minimal algebras. Possibilities are
 1. vector spaces
 2. G -sets
 3. 2-element Boolean algebras (or 2-element fields)
 4. 2-element lattices
 5. 2-element semilattices
- (2) Hobby-McKenzie (1988): partially classified up to polynomial equivalence all finite $\langle \alpha, \beta \rangle$ -minimal algebras, $\alpha \prec \beta$ in $\text{Con}(\mathbb{A})$. Possibilities are
 1. “vector space type”
 2. “ G -set type”
 3. “Boolean algebra type”
 4. “lattice type”
 5. “semilattice type”
- (3) Kiss (1988/1997): classified up to polynomial equivalence all finite $\langle =, A \times A \rangle$ -irreducible algebras. Possibilities are in 5 types, the last 3 of which are the same as in Palfy’s Theorem. The first two are generalizations analogous to the generalization “elementary abelian p -groups generalize to p -groups”.
- (4) Szendrei (1994): classified up to term equivalence all finite strictly simple $\langle =, A \times A \rangle$ -irreducible algebras. Problem splits into 5 types again. Each type splits into 4 subtypes. Some of the 20 resulting classes are empty, some finite, some countably infinite, and some of size continuum.
- (5) Behrisch (2009): classified up to term equivalence all finite subalgebra-primal irreducible algebras.

Covers. (All results refer to polynomial clones.)

- (1) Kearnes-Kiss-Valeriote proved that if \mathbb{A} has a Maltsev polynomial, then it is covered by its $\langle \alpha, \beta \rangle$ -minimal sets, $\alpha \prec \beta$ in $\text{Con}(\mathbb{A})$.
- (2) Kearnes-Szendrei proved that if \mathbb{A} has a $(d + 1)$ -ary NUF, then it is covered by its $\langle R, S \rangle$ -minimal sets, where R and S are d -ary.
- (3) Kearnes-Kiss-Szabo showed that every finite group is covered by its Sylow subgroups.
- (4) Conaway-Kearnes found the nonrefinable, irredundant cover of a finite ring.
- (5) Boercker-Kearnes found the nonrefinable, irredundant cover of any finite algebra in a variety generated by a 2-element algebra.
- (6) Kearnes-Li classified the locally finite varieties whose irreducible sets have a finite size bound.