HISTORY OF MATHEMATICAL IDEAS
MIDTERM

Name:______________________________

You have 50 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

1. Define the following phrases.

(a) *transcendental number.*

A transcendental number is a complex number that is is not a root of a nonzero rational polynomial.

(b) *Fermat prime.*

A Fermat prime is a prime of the form $2^{2^k} + 1$ for some k.

2. Match the person to the event.

Archimedes	Wrote *The Elements*. (Euclid)
Descartes	Proved that if $\alpha \neq 0$ is algebraic, then e^α is transcendental. (Lindemann)
Brahmagupta	Proved that $v - e + f = 2$ for any polygonal dissection of the sphere. (Euler)
Euclid	Proved a sufficient condition for the constructibility of a regular n-gon. (Gauss)
Euler	Introduced a coordinate system for geometry and characterized the constructible numbers via field extensions. (Descartes)
Gauss	Proved that trisection of a general angle cannot be accomplished with straightedge and compass. (Wantzel)
Lindemann	Found a formula for the area of a cyclic quadrilateral. (Brahmagupta)
Wantzel	Found a formula for the area of a parabolic segment. (Archimedes)

3. State and prove the Pythagorean Theorem.

Theorem. If a right triangle has legs a, b and hypotenuse c, then $a^2 + b^2 = c^2$.

Proof. (Many choices here! See `https://www.cut-the-knot.org/pythagoras/`) $\square$

4. Starting with points O and I in the plane, whose distance apart is 1, explain how to construct a segment whose length equals the Golden Ratio. (Hint: the Golden Ration is the positive real number satisfying $\varphi^2 = \varphi + 1$.)

$\varphi = \frac{1+\sqrt{5}}{2}$, so if we can construct a segment AB of length $\sqrt{5}$ we can then concatenate it with a segment BC of length 1, and then bisect AC to get φ. This shows that it is enough to construct $\sqrt{5}$.

Method 1. See the solution to Quiz 3, where the construction of general square roots is explained.

Method 2. Construct a right triangle with legs 1 and 2. The hypotenuse will have length $\sqrt{1^2 + 2^2} = \sqrt{5}$.