

HISTORY (MATH 4820): REVIEW SHEET

I. Math topics.

- (a) Pythagorean theorem.
- (b) Parametrization of conics by rational functions.
- (c) Pythagorean triples.
- (d) Thales' Theorem.
- (e) Euclidean algorithm.
- (f) Commensurable lengths.
- (g) Rational versus irrational numbers.
- (h) The Golden Ratio.
- (i) Bezout's Identity.
- (j) Platonic solids.
- (k) The classification of Platonic solids.
- (l) Euler's Formula.
- (m) Descartes' Theorem about total defect.
- (n) Euler characteristic.
- (o) Gauss-Bonnet Formula.
- (p) Constructibility with straightedge and compass.
 - (i) Planes are associated to fields. Planes closed under Euclidean constructions are associated to Euclidean fields.
 - (iii) A characterization of the field of constructible numbers.
 - (iv) Constructible numbers are algebraic and have minimal polynomial whose degree is a power of 2.
 - (v) Lindemann's Theorem implies that π is not algebraic. Hence circle-squaring is impossible, in general.
 - (vi) A monic polynomial with coefficients in the field rational numbers is a minimal polynomial iff it is irreducible.
 - (vii) $\sqrt[3]{2}$ is not a constructible real number, so cube-doubling is impossible, in general.
 - (viii) The polynomial $x^3 - 3x - 2\cos(\alpha)$ is sometimes irreducible over $\mathbb{Q}[\cos(\alpha)]$. Hence angle-trisection is impossible, in general.
 - (ix) The minimal polynomial of $2\cos(2\pi/n)$ has degree equal to $\frac{1}{2}\varphi(n)$. The regular n -gon is constructible iff this number is a power of 2. This happens exactly when $n = 2^k \cdot p_1 \cdot p_2 \cdots p_r$, where p_1, \dots, p_r are distinct Fermat primes.
- (q) Pell's Equation
- (r) Brahmagupta's Identity (with application to Pell's Equation)
- (s) continued fractions (with application to Pell's Equation)
- (t) method of exhaustion to find areas and volumes
- (u) Quadrature of the Parabola.

(v) modular arithmetic and the Chinese Remainder Theorem.

II. History topics. (Phrased as questions.)

- (a) What approximate date is assigned to the Ishango bone? Why might this bone be interesting?
- (b) What approximate date is assigned to Plimpton 322? What is interesting about this tablet?
- (c) Why is Pythagoras remembered?
- (d) Why is Hippasus remembered?
- (e) Why is Euclid remembered?
- (f) Why is Brahmagupta remembered?
- (g) Why is Descartes remembered?
- (h) Why is Euler remembered?
- (i) Why is Gauss remembered?
- (j) Why is Wantzel remembered?
- (k) Why is Lindemann remembered?

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.