

## History of Mathematical Ideas

### Quiz 7

Name: \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Find all three roots of  $x^3 - 3x - 2 = 0$  using the Cardano Formula.

We have  $x^3 - px - q = x^3 - 3x - 2 = 0$ , so  $p = 3$  and  $q = 2$ . The first root given by the Cardano Formula is  $y_1 = U + V = U + \frac{p/3}{U} = U + \frac{1}{U}$  where

$$U = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}}.$$

The second and third roots have the form  $\omega U + \frac{1}{\omega U}$  and  $\omega^2 U + \frac{1}{\omega^2 U}$ .

For our numbers,  $p = 3, q = 2$ , we have  $U = \sqrt[3]{1 + \sqrt{1^2 - 1^3}} = 1$ , so these expressions reduce to

$$\begin{aligned} y_1 &= 1 + \frac{1}{1} = 2, \\ y_2 &= \omega + \frac{1}{\omega} = \omega + \omega^2 = -1, \\ y_3 &= \omega^2 + \frac{1}{\omega^2} = \omega^2 + \omega = -1. \end{aligned}$$

To summarize, the roots are  $2, -1, -1$ .