History of Mathematical Ideas
Quiz 7

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Find all three roots of $x^3 - 3x - 2 = 0$ using the Cardano Formula.

We have $x^3 - px - q = x^3 - 3x - 2 = 0$, so $p = 3$ and $q = 2$. The first root given by the Cardano Formula is $y_1 = U + V = U + \frac{p/3}{U} = U + \frac{1}{U}$ where

$$U = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}}.$$

The second and third roots have the form $\omega U + \frac{1}{\omega U}$ and $\omega^2 U + \frac{1}{\omega^2 U}$.

For our numbers, $p = 3, q = 2$, we have $U = \sqrt[3]{1 + \sqrt{1^2 - 1^3}} = 1$, so these expressions reduce to

$$\begin{aligned} y_1 &= 1 + \tfrac{1}{1} = 2, \\ y_2 &= \omega + \tfrac{1}{\omega} = \omega + \omega^2 = -1, \\ y_3 &= \omega^2 + \tfrac{1}{\omega^2} = \omega^2 + \omega = -1. \end{aligned}$$

To summarize, the roots are $2, -1, -1$.