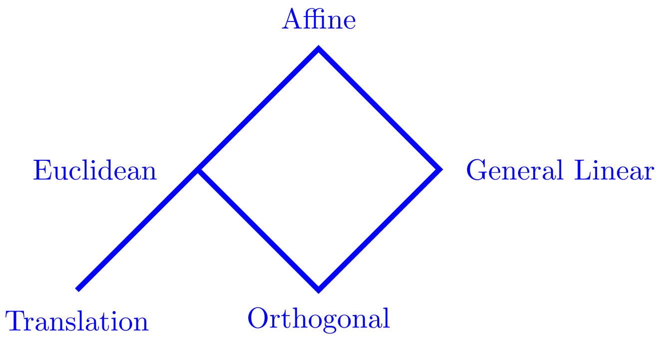
HW 8: solution sketches

These problems are about the following transformation groups of the plane: the translation group, the orthogonal group, the Euclidean group, the general linear group, and the affine group.

(1) Which of the five groups contain which others? (Draw a diagram.)



(2) For each of the five groups, state whether the following properties are preserved by the group: length, angle, area, parallelism of lines, the property that a set of points forms a circle, the property that a set of points forms a triangle, the property that a set of points forms a square.

	Translation	Orthogonal	Euclidean	General Linear	Affine
length	✓	✓	✓	✗	✗
angle	✓	✓	✓	✗	✗
area	✓	✓	✓	✗	✗
parallelism	✓	✓	✓	✓	✓
circularity	✓	✓	✓	✗	✗
triangularity	✓	✓	✓	✓	✓
squareness	✓	✓	✓	✗	✗

(3) Answer this question for each of the five groups: how many types of curves are there which are defined by equations of the form $Ax + By + C = 0$, if curves are considered to be of the same type when a group element can transform one into the other?

(Translation) Infinitely many types. Lines of different slopes are inequivalent under the group.

(Orthogonal) Infinitely many types. Lines with different distances to the origin are inequivalent under the group.

(Euclidean) One type. Any line can be moved to any other using a translation and a rotation.

(Gen. Lin.) Two types. Any line through the origin is equivalent to any other line through the origin, and any line not through the origin is equivalent to any other line not through the origin. But a line through the origin is not equivalent to a line that is not through the origin.

(Affine) One type. Any line can be moved to any other using a translation and a rotation.