

## Practice problems.

These practice problems involve the fact that a matrix has a left inverse if and only if its columns are independent.

Let  $V$  be a subspace of  $\mathbb{R}^k$ . Let  $E$  be a  $k \times n$ -matrix whose columns are independent vectors in  $V$ . Let  $S$  be a  $k \times m$ -matrix whose columns span  $V$ .

- (1) Explain why there is an  $m \times n$ -matrix  $A$  such that  $SA = E$ .
  
  
  
  
  
  
  
  
  
  
- (2) Explain why there is an  $n \times k$ -matrix  $L$  such that  $LSA = LE = I$ .
  
  
  
  
  
  
  
  
  
  
- (3) Conclude that  $n \leq m$ . This means that any linearly independent set of vectors in  $V$  has size less or equal to the size of any spanning set of vectors.