## Practice Problems.

Recall that $T$ is **linear** if (i) $T(x+y) = T(x) + T(y)$, and (ii) $T(r \cdot x) = r \cdot T(x)$ for any scalar $r \in \mathbb{R}$.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2}(\mathbb{R})$ be a $2 \times 2$ matrix. The **transpose** of $A$ is $A^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, which is the matrix obtained from $A$ by reflecting the entries through the main diagonal. (The book uses upper case $T$ in the superscript instead of lower case $t$.) The **trace** of $A$ is $\mathrm{tr}(A) = a + d =$ the diagonal sum. The **determinant** of $A$ is $\det(A) = ad - bc$.

(1) Explain why the transpose function $T(X) = X^t$ is linear.

(2) Explain why the trace function $T(X) = \mathrm{tr}(X)$ is linear.

(3) Explain why the determinant function $T(X) = \det(X)$ is not linear.