Linear Algebra
Quiz 8

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Let $\mathbb{P}(t)$ be the vector space of polynomials in the variable t, with real coefficients, and no bound on degree.

1. Let $F\colon \mathbb{P}(t) \to \mathbb{R}$ be the function that assigns to a polynomial $p \in \mathbb{P}(t)$ its constant coefficient. Explain why F is a linear transformation.

F is defined so that $F(a_0 + a_1 t + \cdots + a_m t^m) = a_0$. We must show that F preserves sums and scaling.

$$\begin{aligned} F((a_0 + \cdots + a_m t^m) + (b_0 + \cdots + b_n t^n)) &= F((a_0 + b_0) + \cdots) \\ &= a_0 + b_0 \\ &= F(a_0 + \cdots + a_m t^m) + F(b_0 + \cdots + b_n t^n). \end{aligned}$$

$$\begin{aligned} F(r(a_0 + \cdots + a_m t^m)) &= F(ra_0 + \cdots) \\ &= ra_0 \\ &= rF(a_0 + \cdots + a_m t^m). \end{aligned}$$

2. Let $G\colon \mathbb{P}(t) \to \mathbb{R}$ be the function that assigns to a polynomial $p \in \mathbb{P}(t)$ its leading coefficient. Explain why G is NOT a linear transformation.

G is defined so that $G(a_0 + a_1 t + \cdots + a_m t^m) = a_m$. G does not preserve sums. For example, here is a sum that is not preserved: $G((1-t) + (1+3t^2)) \neq G(1-t) + G(1+3t^2)$. Check:

$G((1-t) + (1+3t^2)) = G(2 - t + 3t^2) = 3 \neq 2 = (-1) + 3 = G(1-t) + G(1+3t^2)$.

Comment: In (a), to show something is always true, we needed an argument. In (b), to show that something sometimes false, we needed only an example.