**Linear Algebra**
**Quiz 6**

**Name:**______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. (a) Suppose that a square matrix $M$ is partitioned as $\left[\begin{array}{c|c|c} A & 0 & 0 \\ \hline 0 & A & 0 \\ \hline 0 & 0 & -A \end{array}\right]$. Explain why, if $A$ is invertible, then $M^{-1} = \left[\begin{array}{c|c|c} A^{-1} & 0 & 0 \\ \hline 0 & A^{-1} & 0 \\ \hline 0 & 0 & -A^{-1} \end{array}\right]$.

It suffices to observe that the two matrices multiply to $I$:

$$\left[\begin{array}{c|c|c} A & 0 & 0 \\ \hline 0 & A & 0 \\ \hline 0 & 0 & -A \end{array}\right]\left[\begin{array}{c|c|c} A^{-1} & 0 & 0 \\ \hline 0 & A^{-1} & 0 \\ \hline 0 & 0 & -A^{-1} \end{array}\right] = \left[\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & I & 0 \\ \hline 0 & 0 & I \end{array}\right] = I.$$

(b) Find the inverse of $M = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$.

Using part (a) and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ we get $M^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$.

2. What is the dimension of the subspace of all vectors $\mathbf{x} \in \mathbb{R}^n$ whose coordinates sum to zero? (This is the subspace of all $\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ satisfying $x_1 + \cdots + x_n = 0$.)

The answer is $n - 1$.

The matrix equation associated to the system $x_1 + \cdots + x_n = 0$ is

$$\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = 0.$$

The question asks for the dimension of the null space of the $1 \times n$ matrix $A = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$. $A$ is already in reduced row echelon form. The dimension of the column space is 1, since $A$ has one pivot column, so the null space must have dimension $n-1$.

(Or, you could argue this way: the free variables are $x_2, \ldots, x_n$, so $\dim(\mathrm{Nul}(A)) = n - 1$.)