

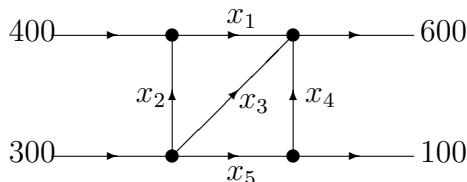
Linear Algebra

Quiz 5

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. The flow of traffic in vehicles per hour through a network of streets is shown in the figure.



- Solve the associated system of linear equations.
- What is the largest possible value for x_3 if all x_i 's must be nonnegative?

To create the system note that the flow into each node must equal the flow out, hence

$$\begin{array}{rclcl} x_1 & -x_2 & & & = 400 \\ x_1 & & +x_3 & +x_4 & = 600 \\ & x_2 & +x_3 & & +x_5 = 300 \\ & & & -x_4 & +x_5 = 100 \end{array}$$

Solving yields

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 700 - x_3 - x_5 \\ 300 - x_3 - x_5 \\ x_3 \\ x_5 - 100 \\ x_5 \end{bmatrix} = \begin{bmatrix} 700 \\ 300 \\ 0 \\ -100 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

which is the answer to (a).

For the answer to (b), note that for flow x_4 to be in the indicated direction we must have $x_5 \geq 100$. Then for flow x_2 to be in the indicated direction we must have $0 \leq x_2 = 300 - x_3 - x_5 \leq 200 - x_3$, so $x_3 \leq 200$. But $x_3 = 200$ can be achieved by $x_1 = 400, x_2 = 0, x_3 = 200, x_4 = 0$ and $x_5 = 100$. Thus, the answer is that 200 is the largest physically possible value for x_3 .

2. Is the $n \times n$ zero matrix invertible? Why or why not?

No. The assumption that the zero matrix is invertible leads to a contradiction. To see this, assume that A is the $n \times n$ zero matrix, and that A^{-1} is its inverse. This assumption yields that

$$I = AA^{-1} = 0.$$

But $I \neq 0$.