Linear Algebra
Quiz 4

Name:____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1.
(a) Define what it means for a set $S \subseteq \mathbb{R}^n$ to be *linearly independent*.

S is linearly independent if it satisfies no nontrivial dependence relation.

(More wordily: S is linearly independent if whenever $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are distinct members of S and $c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k = \mathbf{0}$, then $c_1 = \cdots = c_k = 0$.)

(b) Give an example of a set S that spans $\mathbb{R}^2$ but is not linearly independent.

The simplest example is $S = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{0}\}$.

2. The vector $\mathbf{u}_\alpha = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$ in $\mathbb{R}^2$ has length 1 and points in a direction α radians measured counterclockwise from the x-axis. Write down the standard matrix for the linear transformation r_α that reflects vectors in $\mathbb{R}^2$ through the line that goes through the origin in the direction $\mathbf{u}_\alpha$.

By drawing a picture, you might be able to see that $r_\alpha(\mathbf{e}_1) = \begin{bmatrix} \cos(2\alpha) \\ \sin(2\alpha) \end{bmatrix}$ and $r_\alpha(\mathbf{e}_2) = \begin{bmatrix} \cos(2\alpha - \frac{\pi}{2}) \\ \sin(2\alpha - \frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} \sin(2\alpha) \\ -\cos(2\alpha) \end{bmatrix}$. Hence $[r_\alpha] = \begin{bmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{bmatrix}$.