Linear Algebra
Quiz 3

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Let $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & -2 \\ 0 & 3 & -3 & 6 \end{bmatrix}$.

1. Find the general solution of the associated homogeneous equation. $(A\mathbf{x} = \mathbf{0})$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & -2 \\ 0 & 3 & -3 & 6 \end{bmatrix} \rightsquigarrow \cdots \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

so $x_4 = x_4, x_3 = x_3, x_2 = x_3 - 2x_4, x_1 = -x_3 + x_4$, or

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x_3 + x_4 \\ x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = \boxed{x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}}.$$

2. What is the general solution to the equation $A\mathbf{x} = \mathbf{b}$ which has particular solution $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$?

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \boxed{x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}}$$