Linear Algebra
Quiz 2

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Determine whether $\begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ is in the plane spanned by the columns of $\begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$.

It suffices to show that $\begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ is consistent. Use GE:

$$\left[\begin{array}{cc|c} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 1 & 1 & 4 \\ -2 & 6 & 4 \\ 3 & -5 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{5}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{array}\right], \text{ consistent!}$$

2. Give examples of 2×2 matrices A and C with the properties that $A\mathbf{x} = \mathbf{b}$ is solvable for every $\mathbf{b} \in \mathbb{R}^2$, while $C\mathbf{x} = \mathbf{d}$ is not solvable for at least one $\mathbf{d} \in \mathbb{R}^2$.

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A\mathbf{x} = \mathbf{b}$ is solvable for every $\mathbf{b}$, since $\mathbf{x} = \mathbf{b}$ is a solution.

If $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then $C\mathbf{x} = \mathbf{d}$ is not solvable for any nonzero $\mathbf{d}$.

This problem is related to the theorem discussed on August 31, which says that $A\mathbf{x} = \mathbf{b}$ is solvable for every $\mathbf{b}$ if and only if the RRE form of A has a pivot in every row.