Linear Algebra
Quiz 1

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Consider the following system of linear equations:

$$\begin{array}{rcrcrcrcrcr} 2x_1 & + & 4x_2 & + & x_3 & & & + & x_5 & = & 2 \\ x_1 & + & 2x_2 & & & & & + & x_5 & = & 3 \\ 3x_1 & + & 6x_2 & & & + & x_4 & + & x_5 & = & 4 \end{array}$$

(1) Write down the coefficient matrix. Then write down the augmented matrix.

$$\text{coef.} = \begin{bmatrix} 2 & 4 & 1 & 0 & 1 \\ 1 & 2 & 0 & 0 & 1 \\ 3 & 6 & 0 & 1 & 1 \end{bmatrix}, \quad \text{aug.} = \left[\begin{array}{ccccc|c} 2 & 4 & 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 0 & 1 & 3 \\ 3 & 6 & 0 & 1 & 1 & 4 \end{array}\right]$$

(2) Find the reduced row echelon form of the augmented matrix.

$$\left[\begin{array}{ccccc|c} 2 & 4 & 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 0 & 1 & 3 \\ 3 & 6 & 0 & 1 & 1 & 4 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & 3 \\ 2 & 4 & 1 & 0 & 1 & 2 \\ 3 & 6 & 0 & 1 & 1 & 4 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 0 & 1 & -2 & -5 \end{array}\right]$$

(3) Which variables are free variables and which are basic variables?

x_2 and x_5 are free variables, x_1, x_3 and x_4 are basic variables.

(4) Write the parametric description of the solution set.

$$\begin{array}{rcl} x_1 & = & 3 - 2x_2 - x_5 \\ x_2 & = & \text{free} \\ x_3 & = & -4 + x_5 \\ x_4 & = & -5 + 2x_5 \\ x_5 & = & \text{free} \end{array}$$