Linear Algebra
Quiz 12

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

The real matrix $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}$ has $1+i$ as one of its e-values.

1. Show that $\begin{bmatrix} 1 \\ i-1 \end{bmatrix}$ is an e-vector for A for e-value $1+i$.

$$\begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ i-1 \end{bmatrix} = \begin{bmatrix} 1+i \\ -2 \end{bmatrix} = (1+i) \begin{bmatrix} 1 \\ i-1 \end{bmatrix}$$

2. What is the second e-value? What is an e-vector for this second e-value? (You should be able to answer this without calculation using information from above.)

These can be obtained by conjugating the first e-val and e-vec: $1-i$ and $\begin{bmatrix} 1 \\ -i-1 \end{bmatrix}$.

3. Write down a real matrix S such that $S^{-1}AS$ has block diagonal form where each block is a 1×1 real block or a 2×2 real block of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. (You should be able to answer this without calculation using information from above.)

This matrix is built from the real and imaginary parts of the first e-vec: $S = [\mathrm{Re}(\mathbf{v})\ \mathrm{Im}(\mathbf{v})] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. This is all you need to write, but you can check your answer by multiplying:

$$S^{-1}AS = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$