Linear Algebra
Quiz 11

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Find the e-values and corresponding e-vectors of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

$\chi_A(\lambda) = \det(A - \lambda I) = \det\left(\begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}\right) = \lambda^2 - 5\lambda = \lambda(\lambda - 5)$, so $\lambda = 0, 5$ are the e-values.

$V_0 = \mathrm{Nul}(A - 0I) = \mathrm{Nul}\left(\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\right) = \mathrm{Span}\left\{\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right\}$, so $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is an e-vec for $\lambda = 0$.

$V_5 = \mathrm{Nul}(A - 5I) = \mathrm{Nul}\left(\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\right) = \mathrm{Span}\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$. so $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an e-vec for $\lambda = 5$.

2. Find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$. (Hint: it is possible to use the answer to the previous problem to write down S and D without further calculation.)

$S = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}.$