ALGEBRA 1
HOMEWORK ASSIGNMENT VI

(Turn in underlined problems.)

Read Sections 4.6-6.2.

SECTION	**PROBLEMS**
5.2	**1**
5.5	**1, 6, 14, 18 (Hint: check the holomorph.)**
6.1	**1, 3, 4, 6, 9, 17**
6.2	**1, 4 (just $80, 351$)**

ADDITIONAL PROBLEMS

1. **(Coproducts of abelian groups) Let A and B be abelian groups, written additively. Define *coprojections* $i_A\colon A \to A\times B\colon a \mapsto (a,0)$ and $i_B\colon B \to A\times B\colon b \mapsto (0,b)$. Show that $(A \times B, i_A, i_B)$ is a coproduct of A and B relative to the category of all abelian groups.**

2. Show that the coproduct of $\mathbb{Z}_2$ with $\mathbb{Z}_2$ relative to the category of abelian groups is not the same as the coproduct relative to the category of all groups.

3. Show that

$$\mathbf{Aut}(G_1 \times \cdots \times G_k) \cong \mathbf{Aut}(G_1) \times \cdots \times \mathbf{Aut}(G_k)$$

when the G_i are finite groups of coprime order.

4. For a finite group G, define the *order sequence* of G to be the sequence $(n_1, n_2, \ldots)$, where n_i is the number of elements of G of order i. Show that order sequences determine isotype within the class of finite abelian groups, but not within the class of *all* finite groups.

Challenge Problem! (Prize = fame!) Let G_n be the abelian group presented by $\langle X \mid R \rangle$ where $X = \{x_0, \ldots, x_{n-1}\}$ and R consists of all relations of the form $x_i \cdot x_{i+1} = x_{i+2}$ where the subscripts are taken modulo n. Show that G_n is isomorphic to a group of the form (1) $\mathbb{Z}_m$, (2) $\mathbb{Z}_2 \times \mathbb{Z}_{2m}$, (3) $\mathbb{Z}_m \times \mathbb{Z}_m$, or (4) $\mathbb{Z}_m \times \mathbb{Z}_{5m}$.