ALGEBRA 1

HOMEWORK ASSIGNMENT V

(Turn in underlined problems.)

Read Sections 4.1-4.5, Appendix II.1.

SECTION	**PROBLEMS**
4.1	**6, 8**
4.2	**2, 8, 9, 11, 12**
4.3	**<u>5</u>, 13**
4.4	**<u>18</u>, 19**
4.5	**13, 16, 29, 32, 38**

ADDITIONAL PROBLEMS

1. If G is a simple group, what are the possibilities for the normal subgroup lattice of $G \times G$?

2. Suppose that a category has free objects and coproducts. Show that if $\mathbf{F}(X)$ is a free over X and and $\mathbf{F}(Y)$ is a free over Y, then the coproduct of these two is free over the disjoint union $X \uplus Y$.

3. Under what circumstances will the left regular representation $\rho: G \to S_{|G|}$ of a finite group G have image contained in $A_{|G|}$?

<u>4.</u>

(a) Suppose that G is a finite group and that H is a proper subgroup of G. Show that G is not equal to the union of the conjugates of H.

(b) Suppose that a finite group G acts transitively on a set A. Show that some element of G acts without fixed points.

(c) Explain why both (a) and (b) are false for infinite groups by considering the action of SO(3) on the unit sphere.

5. Suppose that G is a p-group and that $N \triangleleft G$. Show that if $N \neq \{1\}$, then $N \cap Z(G) \neq \{1\}$.

Challenge Problem! (Prize = 1 coin) Find an upper bound on the size and number of finite groups with exactly n conjugacy classes.