ALGEBRA 1
HOMEWORK ASSIGNMENT III

(Turn in underlined problems.)

Read Sections 3.1–3.3.

SECTION	**PROBLEMS**
2.3	**9, 16, 24, 26**
2.4	**7, 10, 11, 13, 14(c)(d), 15, 17**
2.5	**1, 10**
3.1	**1, 9, 12, 14, 38, 39(for any nonabelian group)**

ADDITIONAL PROBLEMS

1. Show that if $S \leq G$ and S has finitely many right cosets in G, then there is a finite set T that is simultaneously a complete set of right coset representatives and a complete set of left coset representatives.

2. There exists a finite group G whose subgroup lattice has maximal chains of different lengths. What is the cardinality of the smallest such group? How many isomorphism types of groups of this smallest cardinality have the desired property?
The following website might help:
`http://hobbes.la.asu.edu/groups/groups.html`

Challenge Problem! (Prize = 1 unusual coin) It is well known and easy to see that a group satisfying the identity $(xy)^2 = x^2y^2$ is abelian. (The identity says $xyxy = xxyy$, so cancelling the left and right factors yields $yx = xy$ for any x and y.) This problem generalizes it by considering other identities of the form

$$\varepsilon_n\colon\ (xy)^n = x^n y^n.$$

Suppose that G is a finite group satisfying identities $\varepsilon_{n_1}, \ldots, \varepsilon_{n_k}$ for some positive integers $n_1, \ldots, n_k$ satisfying

$$\gcd\left(\binom{n_1}{2}, \ldots, \binom{n_k}{2}\right) = 1.$$

Show that G is abelian.