HW worksheeet.

(1) 4.5.32. Suppose that V and W are finite dimensional. Let H be a nonzero subspace of V, and suppose T is a one-to-one linear mapping of V into W. Prove that $\dim(T(H)) = \dim(H)$. If T happens to be a one-to-one mapping *onto* W, then $\dim(V) = \dim(W)$. That is, isomorphic finite dimensional spaces have the same dimension.

(2) 4.6.4. Assume that A is row equivalent to B. Without doing calculations find bases for $\mathrm{Col}(A)$, $\mathrm{Row}(A)$, and $\mathrm{Nul}(A)$, and find the rank and nullity of A.

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(3) 4.6.6. If a 6×3 matrix A has rank 3, find $\dim(\mathrm{Nul}(A))$, $\dim(\mathrm{Row}(A))$, and $\mathrm{rank}(A^t)$.

(4) 4.6.30. Suppose A is $m \times n$ and $\mathbf{b}$ is in $\mathbb{R}^m$. What has to be true about the two numbers $\mathrm{rank}([A|\mathbf{b}])$ and $\mathrm{rank}(A)$ in order for the equation $A\mathbf{x} = \mathbf{b}$ to be consistent?

(5) 4.7.6. Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ and $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$ be bases for a vector space V, and suppose that $\mathbf{f}_1 = 2\mathbf{d}_1 - \mathbf{d}_2 + \mathbf{d}_3$, $\mathbf{f}_2 = 3\mathbf{d}_2 + \mathbf{d}_3$, and $\mathbf{f}_3 = -3\mathbf{d}_1 + 2\mathbf{d}_3$.
(a) Find the change of coordinates matrix from $\mathcal{F}$ to $\mathcal{D}$.
(b) Find $[\mathbf{x}]_{\mathcal{D}}$ for $\mathbf{x}_1 = \mathbf{f}_1 - 2\mathbf{f}_2 + 2\mathbf{f}_3$.

(6) 4.7.8. Find the change of coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, and from $\mathcal{C}$ to $\mathcal{B}$, where

$$\mathcal{B} = \left(\begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right), \quad \mathcal{C} = \left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right).$$

(7) Chap 4 Supplementary: 4. Explain what is wrong with the following discussion: Let $\mathbf{f}(t) = 3 + t$ and $\mathbf{g}(t) = 3t + t^2$, and note that $\mathbf{g}(t) = t\mathbf{f}(t)$. Then $\{\mathbf{f}(t), \mathbf{g}(t)\}$ is linearly dependent because $\mathbf{g}$ is a multiple of $\mathbf{f}$.

(8) Chap 4 Supplementary: 10. Let S be a maximal linearly independent subset of a vector space V. That is, S has the property that if a vector not in S is adjoined to S, then the new set will no longer be linearly independent. Prove that S must be a basis for V.

(9) Chap 4 Supplementary: 12. Show that the rank of AB cannot exceed the rank of A or the rank of B, by doing parts (a) and (b). ... (See the book!)
(a)
(b)

(10) Extra: 1. Suppose that $\mathbb{V} \leq \mathbb{R}^3$ is spanned by $\mathcal{B} = \left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right)$ and $\mathbb{W} \leq \mathbb{R}^3$ is spanned by $\mathcal{C} = \left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix} \right)$. Find bases for $\mathbb{V} + \mathbb{W}$ and $\mathbb{V} \cap \mathbb{W}$.