SET THEORY (MATH 4730)

SUMMARY OF TOPICS FROM 8/26/13-10/7/13

I. Axiomatic Set Theory
   (a) Axioms 1-8.
   (b) Valid constructions of new sets (pairing, union, power set, comprehension, intersection).
   (c) Russell's Paradox.
   (d) Empty set, successor of a set, inductive sets.

II. Relations and Functions
   (a) Ordered pairs, product sets.
   (b) Relations.
   (c) Functions, equivalence relations, partitions, kernel, image.
   (d) Classes and class functions.
   (e) Ordered sets, linear orders, well-orders.

III. Natural numbers
   (a) Definition of ω.
   (b) Ordinary and strong induction.
   (c) Induction over $(X, <)$ is valid iff $(X, <)$ is well founded.
   (d) ω is well-ordered.
   (e) Arithmetic of ω.
   (f) Recursion Theorem.
   (g) Parametrized Recursion and Course of Values Recursion.

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.

Sample Problems.

(1) Explain why the intersection of two sets is a set.

(2) Prove or disprove:
   (a) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.
   (b) $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

(3) What is a function? (Give the definition.)

(4) What is the kernel of the successor function, considered as a function from ω to ω?

(5) Explain why induction is a valid form of proof. (Your explanation should make use of the fact that ω is a subset of every inductive set.)

(6) Show that $m + n = 0$ implies $m = n = 0$ for natural numbers m and n. Then show that $m + n = 1$ implies that m and n are 0 and 1 in some order.

(7) Given a recursive definition of a function from ω to A, show that at most one function can satisfy the definition.