Set Theory
Quiz 9

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Write down the recursive definitions of ordinal sum and product.

$$\begin{aligned} \alpha + 0 &= \alpha \\ \alpha + S(\beta) &= S(\alpha + \beta) \\ \alpha + \beta &= \sup_{\gamma<\beta}(\alpha + \gamma) \qquad \text{if } \beta \text{ is limit} \end{aligned}$$

$$\begin{aligned} \alpha \cdot 0 &= 0 \\ \alpha \cdot S(\beta) &= \alpha \cdot \beta + \alpha \\ \alpha \cdot \beta &= \sup_{\gamma<\beta}(\alpha \cdot \gamma) \qquad \text{if } \beta \text{ is limit} \end{aligned}$$

2. Which ordinal number is larger? (All arithmetic operations refer to ordinal arithmetic.)

(a) $2 + \omega$ or $\omega + 2$?

$2 + \omega = \omega < \omega + 2$

(b) $2 \cdot \omega$ or $\omega \cdot 2$?

$2 \cdot \omega = \omega < \omega + \omega = \omega \cdot 2.$