

**Set Theory**  
**Quiz 5**

**Name:** \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Prove that addition is cancellative: for all  $m, n, k$  in  $\omega$  we have

$$m + k = n + k \quad \text{implies} \quad m = n.$$

Let  $P(k)$  be: " $\forall m \forall n ((m + k = n + k) \rightarrow (m = n))$ "

$P(0)$  holds: Assume that  $m + 0 = n + 0$ .

$$\begin{array}{ll} m + 0 &= n + 0 \\ m &= n \end{array} \quad \begin{array}{l} \text{Assumption} \\ \text{(IC, +)} \end{array}$$

$P(k) \rightarrow P(S(k))$  holds: Assume that  $P(k)$  holds and that  $m + S(k) = n + S(k)$ .

$$\begin{array}{ll} m + S(k) &= n + S(k) & \text{Assumption} \\ S(m + k) &= S(n + k) & \text{(RR, +)} \\ m + k &= n + k & \text{Successor is injective} \\ m &= n & \text{Inductive hypothesis, } P(k) \end{array}$$

Hence  $P(S(k))$  holds.