

**Set Theory**  
**Quiz 4**

**Name:** \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show that every nonzero natural number is the successor of some natural number.

Let's use induction to prove  $P(x)$ : " $(x = 0) \vee \exists k(x = S(k))$ ".

Basis:  $P(0)$  holds trivially.

Inductive step: Assume that  $P(n)$  holds for some natural number  $n$ . We must prove that  $P(S(n))$  holds. That is, we must show that if  $S(n)$  is nonzero, then  $S(n)$  is the successor of some natural number. But of course this is true, because  $S(n)$  is the successor of  $n$ . (That is, to prove that  $P(x)$  holds for  $x = S(n)$ , take  $k = n$ .)