Set Theory
Quiz 10

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Prove the following statement by tranfinite induction: If α and β are ordinals, then $\beta \leq \alpha + \beta$.

We prove "$\forall\alpha(\beta \leq \alpha + \beta)$" by induction on β.

$\beta = 0$: $\forall\alpha(0 \leq \alpha + 0 = \alpha)$ holds since 0 is the least ordinal.

$\beta = S(\gamma)$: for any α we have $\gamma \leq \alpha + \gamma$ by induction. Since the successor operation is order-preserving we derive the red inequality in

$$\beta = S(\gamma) \leq S(\alpha + \gamma) = \alpha + S(\gamma) = \alpha + \beta.$$

β is limit: for any α we have $\gamma \leq \alpha + \gamma$ for all $\gamma < \beta$ by induction. Since $\sup_{\gamma<\beta}$ is order-preserving we derive the red inequality in

$$\beta = \sup_{\gamma<\beta}(\gamma) \leq \sup_{\gamma<\beta}(\alpha + \gamma) = \alpha + \beta.$$